Year 9
Knowledge Organiser

Year 9- Indices and Standard Form

The Laws of Indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

Laws of Indices- Examples

Simplify each of the following:

1) $a^6 \times a^4 = a^{6+4}$
$= a^{10}$

2) $a^6 \div a^4 = a^{6-4}$
$= a^2$

3) $(a^6)^4 = a^{6\times 4}$
$= a^{24}$

4) $(a^4)^3 = a^{4\times 3}$
$= a^{12}$

5) $\frac{5^2 \times 5^6}{5^4} = \frac{5^8}{5^4}$
$= 5^{8-4}$
$= 5^4$

6) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Standard Form

We use standard form to write a very large or a very small number.

Must be $\times\ 10$

$$a \times 10^b$$

Must be $1 \leq a < 10$

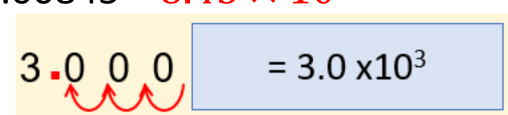
b is an integer

Standard Form- Examples

Write the following in **standard form**:

1) $3000 = 3 \times 10^3$

2) $0.00845 = 8.45 \times 10^{-3}$



Key Words

Indices
Reciprocal
Rounding
Estimate
Significant Figure
Standard Form

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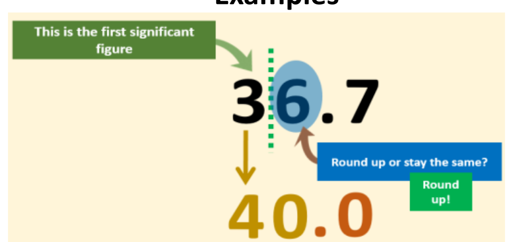
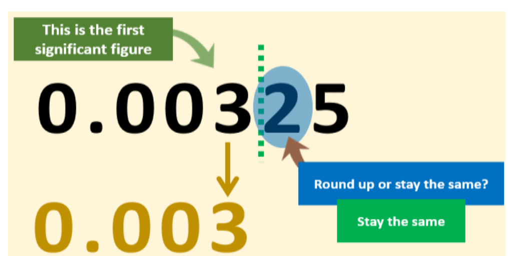
102 – 110, 122-123, 130,

Tip- Estimating

When you are asked to 'estimate', you only need an approximate answer. To find this you **round** the numbers involved to **1 significant figure**.

Rounding to 1 significant Figure- Examples





Estimating- Examples

$78.9 \div 7.8$

$80.0 \div 8.0$

10

$$\frac{41.3 \times 8.1}{2.37}$$

$$\frac{40 \times 8}{2} = \frac{320}{2} = 160$$

Year 9- Expressions and Formulae

Key Concepts

A **formula** is a rule written using letters and symbols, where one letter equals an **expression** of other letters.

When **substituting** a number into an expression, replace the letter with the given value.

Changing the Subject of a formula involves the same process as solving equations.

Substituting- Examples

1) Find the value of $3x + 2$ when $x = 5$

$(3 \times 5) + 2 = 17$

2) Where $A = b^2 + c$, find A when b = 2 and c = 3

$A = 2^2 + 3$

$A = 4 + 3$

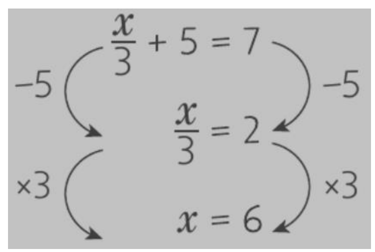
$A = 7$

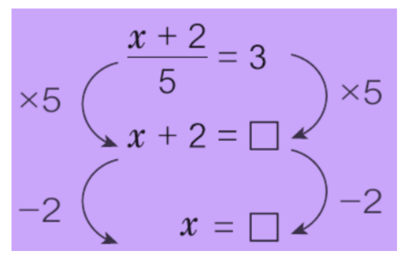
Tip

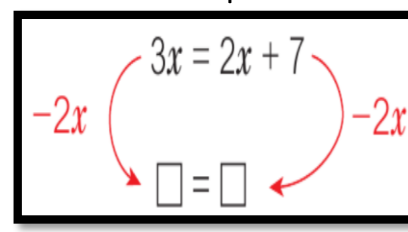
Don't forget that "3x" means "3 lots of x", so you have to multiply the co-efficient by the number you substitute in.

Solving Equations - Examples

To solve equations, you need to get the unknown on it's own. To do this, do the inverse to whatever is with the x. Do this to both sides of the equations.







Key Words

Substitute
Equation
Formula
Expression
Expand

Key Concepts

Expanding brackets means to multiply. The process for this if different for single and double brackets.

Expanding Brackets- Examples

Expand and simplify where appropriate

1) 7 (3 + a) = 21 + 7a

2) f(f + 6) = f^2 6f

Expanding Double Brackets- Example

$(x + 6)(x + 4)$

First terms: $x \times x = x^2$

Outside terms: $x \times 4 = 4x$

Inside terms: $6 \times x = 6x$

Last terms: $6 \times 4 = 24$

$x^2 + 4x + 6x + 24$

$x^2 + 10x + 24$

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160-164, 177-184, 780-787, 155, 280-282

Year 9- Dealing with Data

Key Concepts

A **sample** is a smaller part of a population intended to show what the whole population is like.

Primary Data is data you collect yourself.

Secondary Data is collected by someone else.

Frequency Table- Examples

Frequency tables are a good way to show data. There are two types- **Standard** and **Grouped**.

The type you use depends on the type of data you have collected

Number of Pets	Frequency
0	3
1	5
2	4
3+	3

Height	Frequency
120≤x<130	8
130≤x<140	12
140≤x<150	10
150≤x<160	9
160≤x<190	11
	50

Estimating Mean from a Frequency Table

Height	Frequency	Midpoint (m)	m × f
120≤x<130	8	125	1000
130≤x<140	12	135	1620
140≤x<150	10	145	1450
150≤x<160	9	155	1395
160≤x<190	11	175	1925
	50		7390

$\frac{7390}{50} = 147.8cm$

Key Words

Data

Sample

Averages

Line of best fit

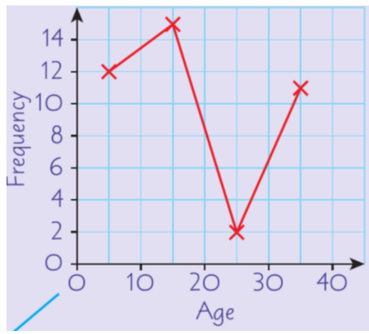
Stem and Leaf

Line Graphs- Example

Age, a	Frequency	Midpoint
$0 \leq a < 10$	12	5
$10 \leq a < 20$	15	15
$20 \leq a < 30$	2	25
$30 \leq a < 40$	11	35

First work out the midpoint of each class.

Plot each frequency against the midpoint age.



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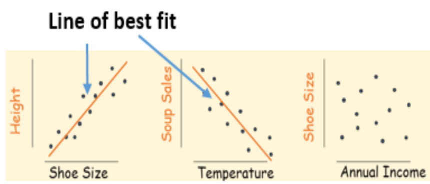
392-393, 416-418, 453-454, 430-433

Scatter Graph - Examples



- Scatter graph 1 shows a positive correlation.
- Scatter graph 2 shows a negative correlation.
- Scatter graph 3 shows no correlation.

Stem and Leaf Diagram- Example

Stem and Leaf diagrams allow you to list data. When you have more than 1 of them you can easily compare the **distribution** of the data to help you anaylse.

Temperature in different cities

0	3	5	6	6		
1	2	4	5	5	6	9
2	5	6	7	7	7	
3	0	1				

Key: 1 | 6 means 16°C

Year 9- Multiplicative Reasoning

Key Concepts

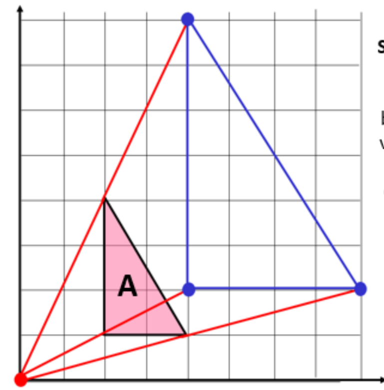
An **enlargement** changes the size of a shape. It can get larger or smaller, depending on the **scale factor.**

Two quantities are in **direct proportion** if they increase (or decrease) by the same ratio. e.g. Number of pens and weight of pens.

Two quantities are **inversely proportional** if one increases by the same ratio as the other decreases.

Enlargement- Example

Enlarge shape A, scale factor 2, centre (0, 0).

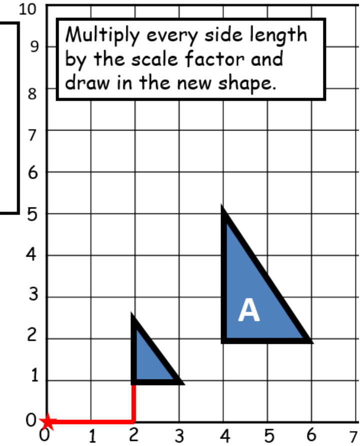


Scale factor 2 - Double the distance between each vertex and the centre of enlargement.

Fractional Scale Factors

Transform triangle A by the enlargement:

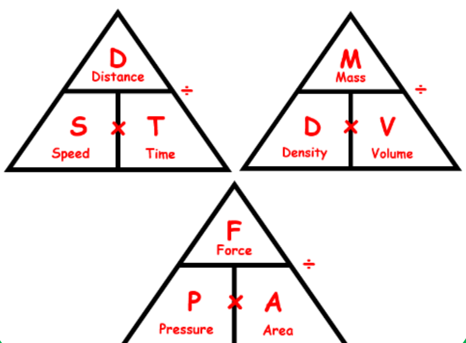
Scale factor $\frac{1}{2}$ from centre (0, 0)



Key Words

Scale Factor
Enlargement
Percentage
Compound
Proportion

Compound Measures



Formula

$$\text{percentage change} = \frac{\text{actual change}}{\text{original amount}} \times 100$$

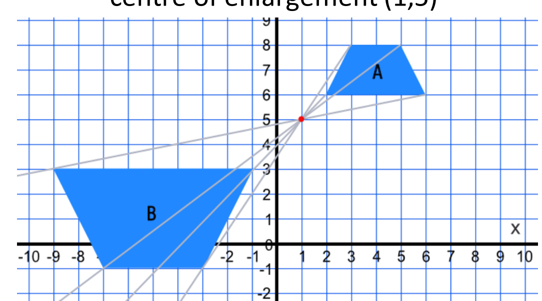
Tip

Percentage profit/loss is the percentage change between cost price and selling price. Use the same formula as percentage change.

Negative Scale Factors

When the scale factor is negative, your new shape will be in the other direction from the centre of enlargement.

Multiply the distance between the original vertices and the centre of enlargement by the scale factor.
e.g Enlarge shape A by scale factor -2 about the centre of enlargement (1,5)



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642-647, 90, 97, 719-737, 339-342

Direct/Inverse Proportion- Examples

Direct proportion:

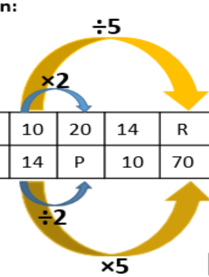
Value of A	32	P	56	20	72
Value of B	20	30	35	R	45

Ratio constant: $20 \div 32 = \frac{5}{8}$

From A to B we will multiply by $\frac{5}{8}$.
From B to A we will divide by $\frac{5}{8}$.

$P = 30 \div \frac{5}{8} = 48$ $R = 20 \times \frac{5}{8} = 12.5$

Inverse proportion:



Value of A	10	20	14	R	28
Value of B	14	P	10	70	5

P = 7 R = 2

Year 9- Constructions

Key Concepts

A **construction** is an accurate drawing. It has to be exact. Using rulers, compasses and protractors properly is crucial in constructions.

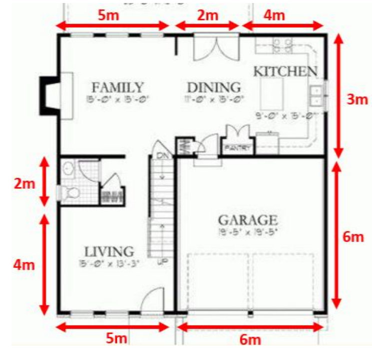
Scales are used in maps and other drawings. They allow us to fit very big things, or very small things on our page.

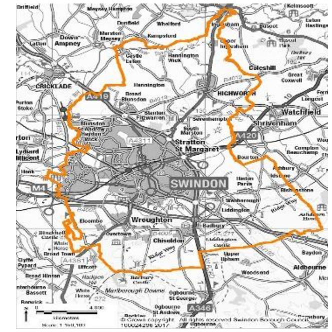
Scales- Example

A scale of

1 cm : 2m

...means for every 1cm measured on a drawing this represents 2m in real-life.

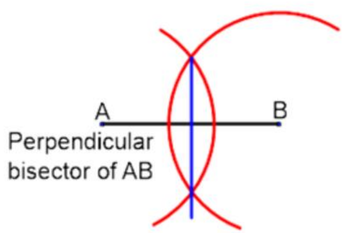


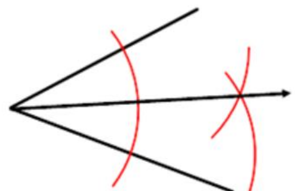


Key Words

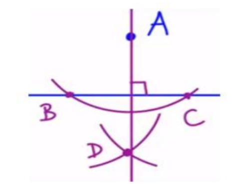
Scale
Construct
Perpendicular
Bisector

Basic Construction- Examples

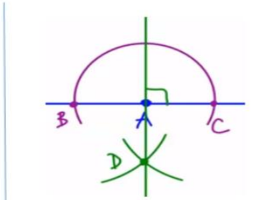




Angle Bisector



Through a given point (A) not on the line (BC)



Through a given point (A) on the line (BC)

Tip

To find the real life size from a scale drawing, multiply the distance on your page by the number on the scale.

But make sure your units are the same first!

Key Concept

There are three types of triangle you need to be able to construct:

SAS- **S**ide **A**ngle **S**ide
ASA- **A**ngle **S**ide **A**ngle
SSS-**S**ide **S**ide **S**ide

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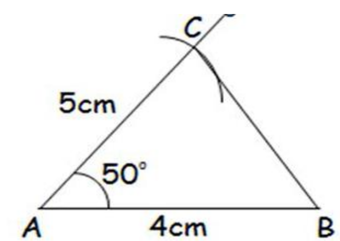
660-663, 864-868, 683

Triangle Construction- Examples

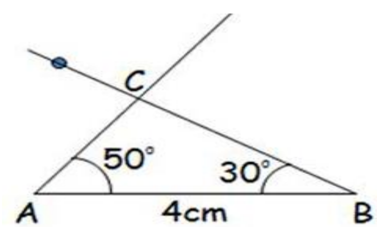
SAS- Side Angle Side

You need a protractor, compass and ruler for these.



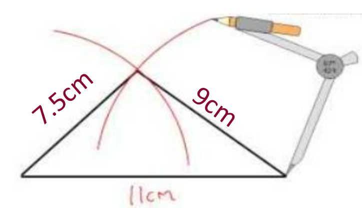
ASA- Angle Side Angle

You need a protractor, compass and ruler for these.



SSS-Side Side Side

You just need a compass and ruler for these.



Year 9- Sequences, Inequalities, Equations and Proportion

Key Concept- Sequences

Arithmetic or linear sequences increase or decrease by a common amount each time.
Geometric series has a common multiple between each term.
Quadratic sequences include an n^2. It has a common second difference.
Fibonacci sequences are where you add the two previous terms to find the next term.

Linear/arithmetic sequence:

-3 +3 +3 +3 +3

+1 4 7 10 13 16…..

a) State the nth term

$3n + 1$

Difference ↗ ↖ The 0th term

b) What is the 100th term in the sequence?

$3n + 1$

$3 \times 100 + 1 = 301$

c) Is 100 in this sequence?

$3n + 1 = 100$

$3n = 99$

$n = 33$

Yes as 33 is an integer.

Key Concept- Proportion

Variables are **directly proportional** when the **ratio is constant** between the quantities.

Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing.**

Proportion - Examples

If Y is **directly proportional** to x. Where Y =6 when x = 3.	If Y is **inversely proportional** to x. Where Y =5 when x = 4.
Y = kx	Y = k/x
6 = 3k	5 = k/4
2 = k	20 = k
Y = 2x	Y = 20/x

Key Concept- Inequalities

Inequalities show the **range** of numbers that satisfy a rule.

$x < 2$ means x is less than 2
$x \leq 2$ means x is less than or equal to 2
$x > 2$ means x is greater than 2
$x \geq 2$ means x is greater than or equal to 2

On a **number line** we use circles to highlight the key values:

is used for less/greater than
is used for less/greater than

Key Words

Nth Term
Inequality
Equation
Proportion

Inequalities- Example

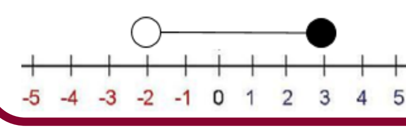
a) State the values of n that satisfy:

$-2 < n \leq 3$

Cannot be equal to 2 ↗ ↖ Can be equal to 3

-1, 0, 1, 2, 3

b) Show this inequality on a number line:



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198, 247, 264-268, 177-184, 339-347

Geometric sequence e.g. ×3 ×3 ×3

4 12 36 108…

Quadratic sequence e.g. $n^2 + 4$ Find the first 3 numbers in the sequence

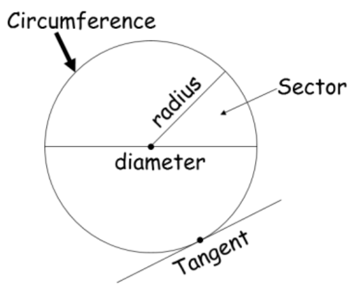
First term: $1^2 + 4 = 5$

Second term: $2^2 + 4 = 8$

Third term: $3^2 + 4 = 13$

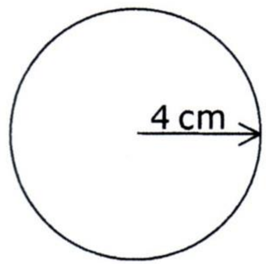
Year 9- Circles, Pythagoras and Prisms

Key Concept- Circles



Formula/Examples- Circles

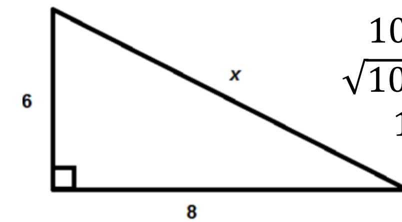
Find the area and circumference to 2dp.



$$Circumference = \pi \times d$$
$$= \pi \times 8 = 25.13cm$$

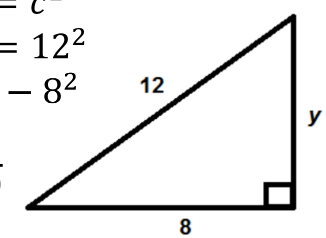
$$Area = \pi \times r^2$$
$$= \pi \times 4^2 = 50.27cm^2$$

Examples- Pythagoras' Theorem



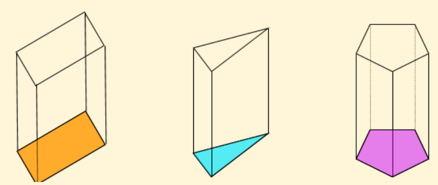
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = x$$
$$10 = x$$

$$a^2 + b^2 = c^2$$
$$y^2 + 8^2 = 12^2$$
$$y^2 = 12^2 - 8^2$$
$$y^2 = 80$$
$$y = \sqrt{80}$$
$$y = 8.9$$

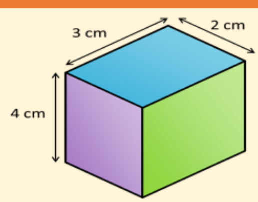


Key Concept- Prisms

Volume of a prism = area of cross-section x vertical height



Surface Area is the total area of all the faces of a 3D shape



Key Concept- Pythagoras

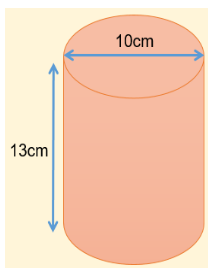
Pythagoras' theorem and basic trigonometry both only work with **right angled triangles**.

Pythagoras' Theorem – used to find a missing length when two sides are known

$$a^2 + b^2 = c^2$$

c is always the hypotenuse (longest side)

Example- Cylinders



Volume = π x 5^2 x 13

= 325π

= 1021.0 cm^3 (to 1 dp)

Key Concept- Errors and Bounds

Upper Bound: The lowest number that rounds to the number given

Lower Bound: The lowest number that round to the next number

Example: The length of a screw is 6cm to the nearest cm.

Lower Bound = 5.5cm

Upper Bound = 6.5cm

Error Interval: 5.5 ≤ length ≤ 6.5

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534-547; 498-501; 570-571; 137-139; 572-574

Year 9- Graphs

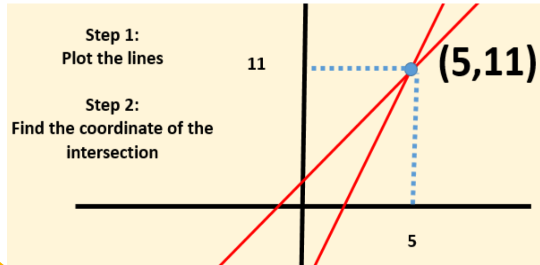
Key Concept- Straight Lines

$y = mx + c$

m is the gradient, or the slope of the graph

c is the y-intercept, or where the graph cuts the y-axis

Key Concept- Simultaneous Equations

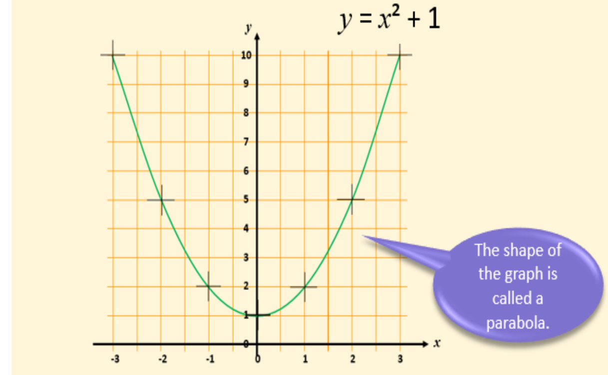


Example- Plotting Quadratics

$y = 0^2 + 1$ $y = 3^2 + 1$

X	-3	-2	-1	0	1	2	3
Y	10	5	2	1	2	5	10

$y = x^2 + 1$



The shape of the graph is called a parabola.

Example- Straight Lines

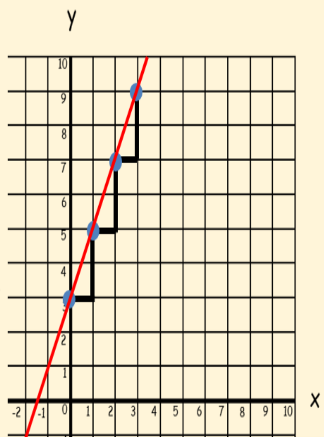
Sketch

$y = 2x + 3$

Start at 3 on the y-axis.

For every 1 across, go up 2.

Join with a straight line.



Remember, you don't need the black lines attached to the red line- that's just to show you how to work out the gradient.

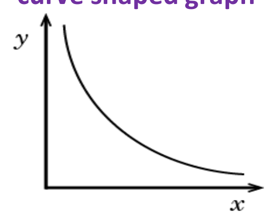
Key Concept- Non-linear graphs

Inverse proportion:

÷5 ×2 ÷2 ×5

Value of A	10	20	14	R	28
Value of B	14	P	10	70	5

You can then plot this as a graph. It will create a curve shaped graph



Key Words

- Linear
- Non-linear
- Gradient
- Y-intercept
- Quadratic
- Inverse Proportion

Examples- ax+by=c

6y - 12x = 30 (+12x)
6y = 30 + 12x (÷6)
y = 5 + 2x
y = 2x + 5

3x + y = 18 (-3x)
y = 18 – 3x
y = -3x +18

Once the subject is Y, we can plot the same as we do with Y=Mx+C

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204-210; 217-219; 248; 251; 252

Key Concept- ax+by=c

Sometimes, an equation won't be in y = mx + c format. But we can make it so by rearranging to make Y the subject

Or you can substitute x=0 into the equation to find the y-intercept, then substitute y=0 to find the x-intercept.

Year 9- Probability

Formula- Probability of an outcome

Probability of an outcome = $\dfrac{\text{the number of ways the outcome can happen}}{\text{the number of all possible outcomes}}$

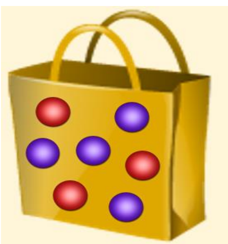
Key Concept- Mutually Exclusive Events

The probabilities of mutually exclusive events add to 1

The probability of flipping heads is $\frac{1}{2}$

The probability of flipping a tails is $\frac{1}{2}$ Total: $\frac{1}{2}+\frac{1}{2}=1$

Example- Probability



P(Purple) = $\frac{4}{7}$

P(Red) = $\frac{3}{7}$

P(Green) = $\frac{0}{7}$

Key Concept- Theoretical vs. Experimental Probability

Theoretical Probability:	Experimental Probability:
The probability you would expect. E.g. Flipping a coin and getting tails = ½	What actually happens in reality e.g. when you flip a coin you don't get tails exactly half of the time

Example- Expected Outcomes

We can use experimental probability to make predictions about what we expect to happen in future.

The probability of a blue car is 0.2,

- If 10 cars go past I expect to see 0.2 x 10= 2 blue cars
- If 40 cars go past I expect to see 0.2 x 40= 8 blue cars
- If 55 cars go past I expect to see 0.2 x 55= 11 blue cars

Key Words

Probability
Mutually Exclusive
Experimental
Bias
Sample Space
Venn Diagram

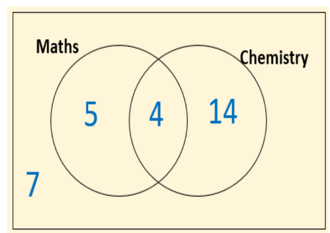
Example- Venn Diagrams

We can use Venn diagrams to calculate probabilities.

P(Study both) = $\frac{4}{30}$

P(Study Maths) = $\frac{9}{30}$

P(Study neither) = $\frac{7}{30}$



Example- Two-way tables

	English	Maths	Sci	Total
Girls	20	13	17	50
Boys	18	15	23	56
Total	38	28	40	106

P(student chose Maths) = 28/106

P(girl chose science) = 17/40

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350-356; 370-380; 670-673; 422-424

Key Concept- Probability Notation

When we are talking about probability, we use the following notation:

P(x) = **'x' represents whatever it is we're talking about**

e.g. P(heads) means 'The Probability of flipping a heads"

P (heads) = ½ or 0.5

Year 9 Trigonometry

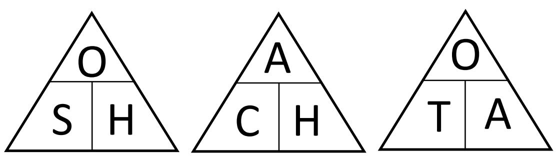
Key Concept

Basic trigonometry only works with **right angled triangles** (like Pythagoras' Theorem)

Formulae

Basic trigonometry SOHCAHTOA – used to find a missing side or an angle

Instead of memorizing all 3 of the trigonometric ratios, remember these formula triangles instead.

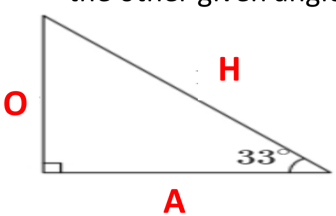


Key Concept

H = Hypotenuse (longest side, opposite the right angle)

O = Opposite (opposite the other given angle)

A = Adjacent (in between the right angle and the other given angle)



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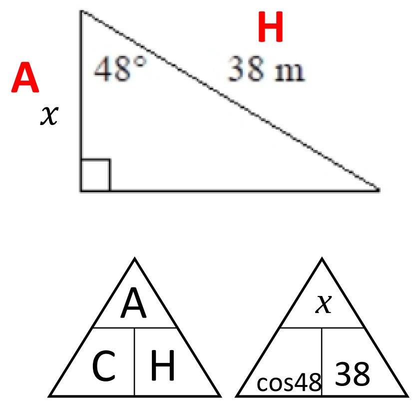
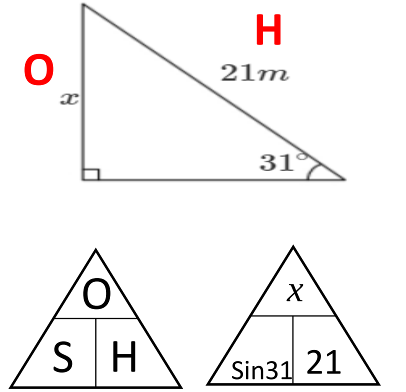
498-499, 509-515

SOHCAHTOA to find Lengths- Examples



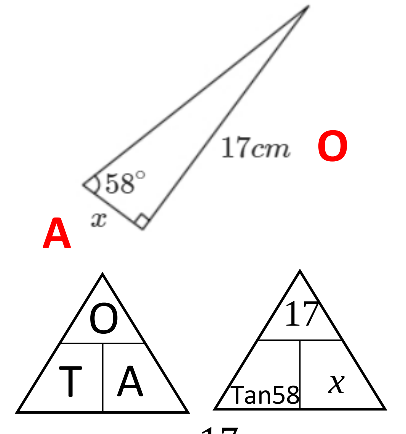
$$\cos 48 = \frac{x}{38}$$

$$x = 38 \times \cos 48 = 25.4m$$



$$\sin 31 = \frac{x}{21}$$

$$x = 21 \times \sin 31 = 10.8m$$

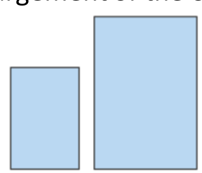


$$Tan\ 5 = \frac{17}{x}$$

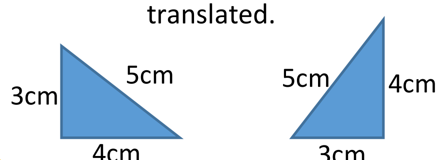
$$x = \frac{17}{Tan\ 58} = 10.6\text{m}$$

Key Concept

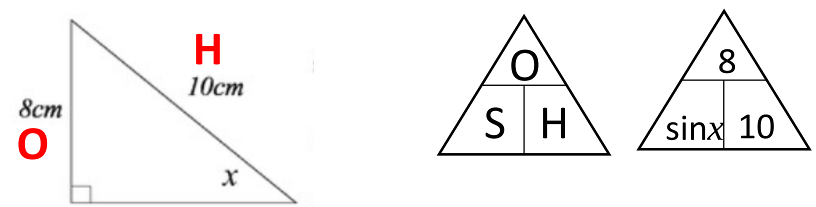
Two shapes are **similar** if one is an enlargement of the other.



Two shapes are **congruent** if they are *exactly* the same shape. They may be reflected, rotated or translated.



SOHCAHTOA to find Angles- Example



Remember- angles are shifty! (Press shift on your calculator to get sin^{-1})

$$\sin x = \frac{8}{10}$$

$$x = \sin^{-1}\left(\frac{8}{10}\right) = 53.1^o$$